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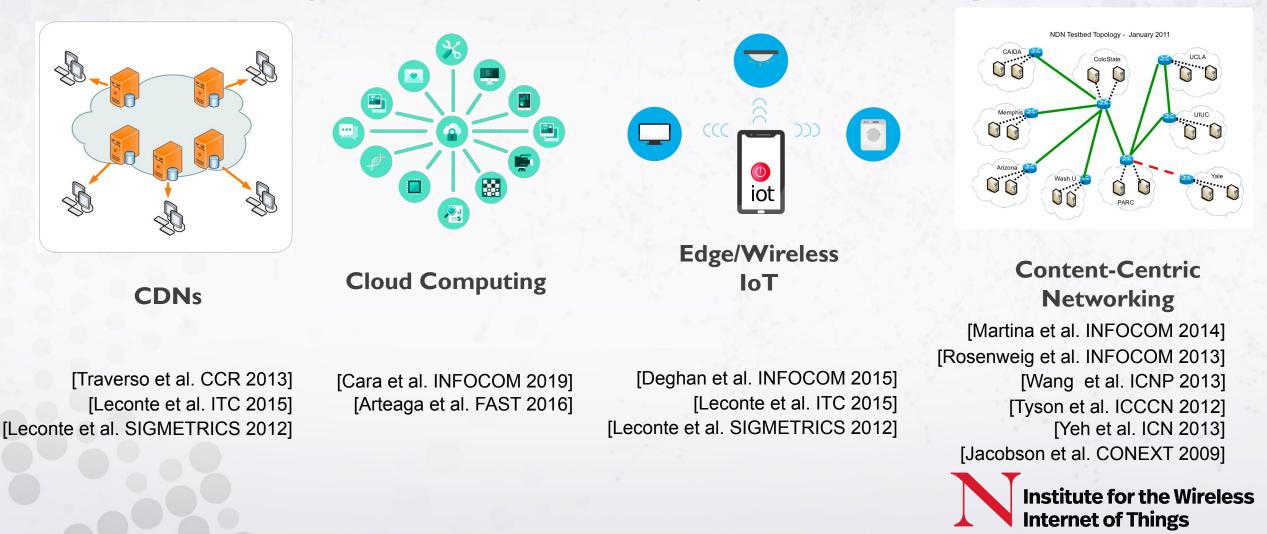
Cache Networks with Optimality Guarantees

Stratis Ioannidis Department of Electrical and Computer Engineering Northeastern University

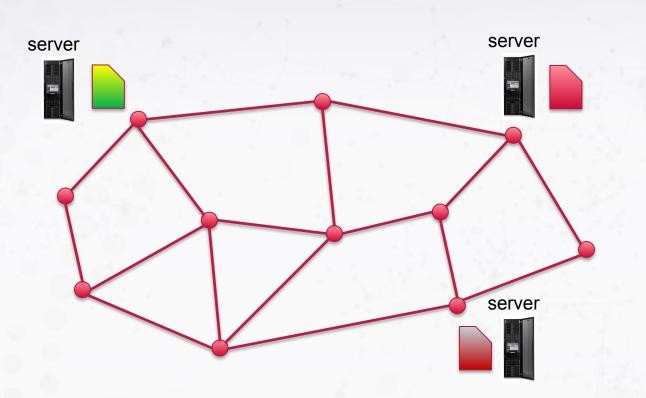
Joint work with Khashayar Kamran, Yuezhou Liu, Yuanyuan Li, Qian Ma, Milad Mahdian, Armin Moharrer, Tareq Si Salem, Giovanni Neglia, and Edmund Yeh

Motivation

Caching and object allocation problems are ubiquitous

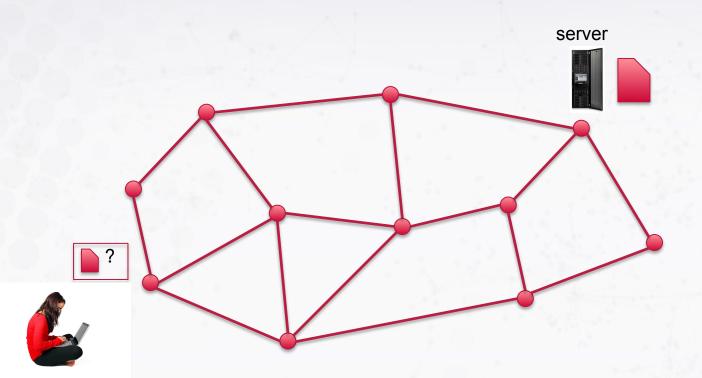


at Northeastern



Designated servers in the network store content items (e.g., files, file chunks).

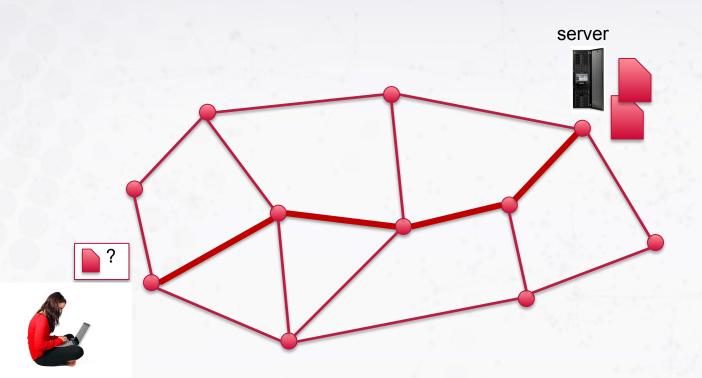




User

Nodes generate **requests** for content items

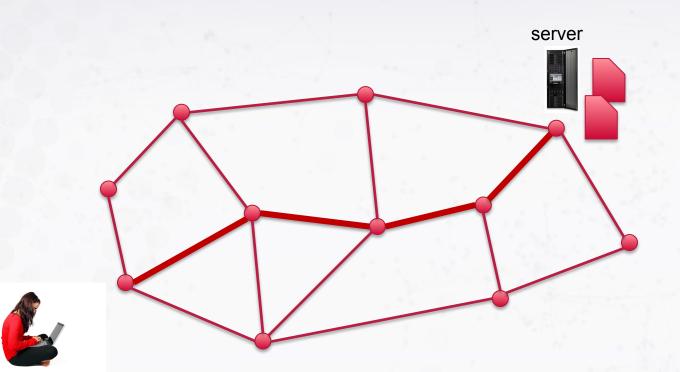




User

Requests routed towards a designated server

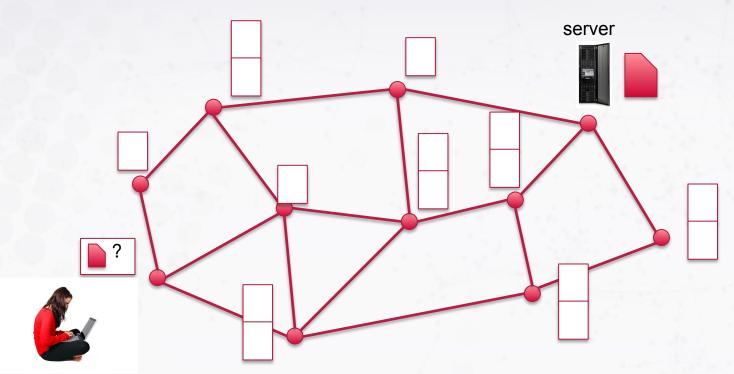




User

Responses routed over **reverse** path

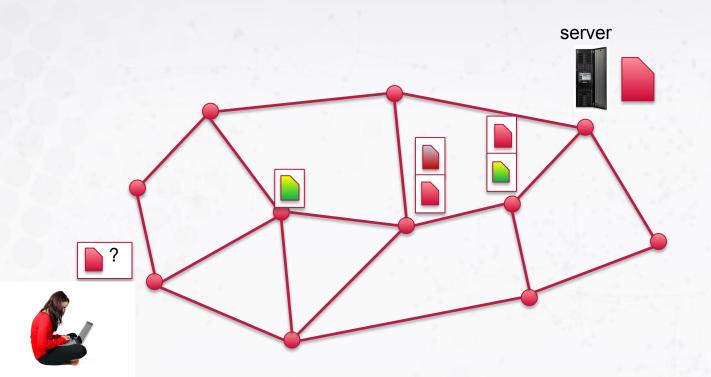




User

Nodes have **caches** with finite capacities

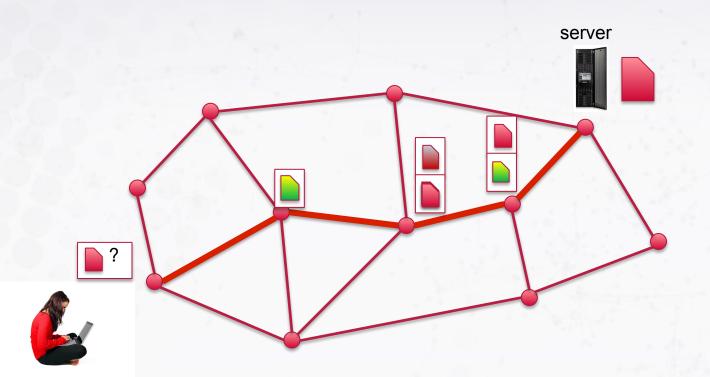




User

Nodes have **caches** with finite capacities





User

Requests terminate early upon a cache hit

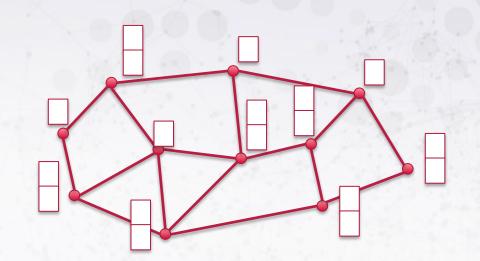


Cache Network Problems

Cache Networks: nodes can store **content.**

- Optimize caching decisions
- □ ...plus:
 - **Routing**
 - **Scheduling/service allocation**
 - **Admission control**
 - **D** ...
- Minimize delays or transfer costs, maximize throughput or utility, incorporate fairness, study stability ...

Distributed, **adaptive** algorithms



Much, much harder, because caching is combinatorial!!!



Our Research Contributions

Distributed, adaptive, algorithms optimizing **caching** decisions

Stochastic requests

Adversarial requests/no-regret setting

□ Joint optimization of caching **and** routing

Queuing Models

Kelly cache networks

Cache networks with counting queues

Stability/admission control

Fair caching networks

[I. and Yeh, SIGMETRICS 2016/ToN 2018]

[Li, Si Salem, Neglia, and I., SIGMETRICS 2022]

[I. and Yeh, ICN 2017/JSAC 2018] [Li, Si Salem, Neglia, and I., SIGMETRICS 2022]

[Mahdian, Moharrer, I., and Yeh, INFOCOM 2019/ToN 2020]

[Li and I., INFOCOM 2020/ToN 2021]

[Kamran, Moharrer, I., and Yeh, INFOCOM 2021]

[Liu, Li, I., and Yeh, Performance 2020]

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□ Cache network optimization

□ Jointly optimizing caching and routing

Introducing queues



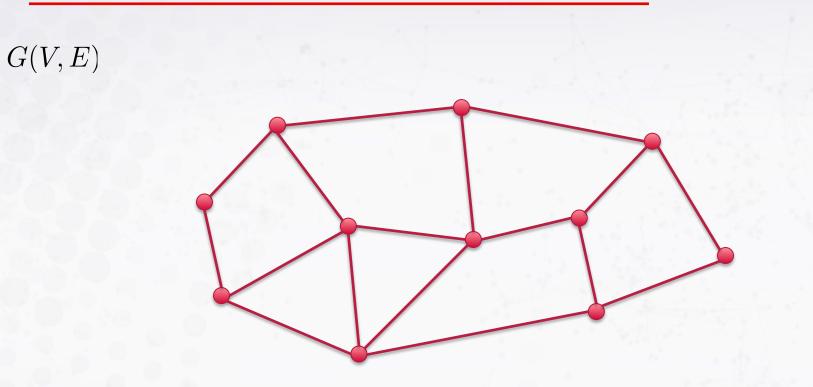


□ Cache network optimization

□ Jointly optimizing caching and routing

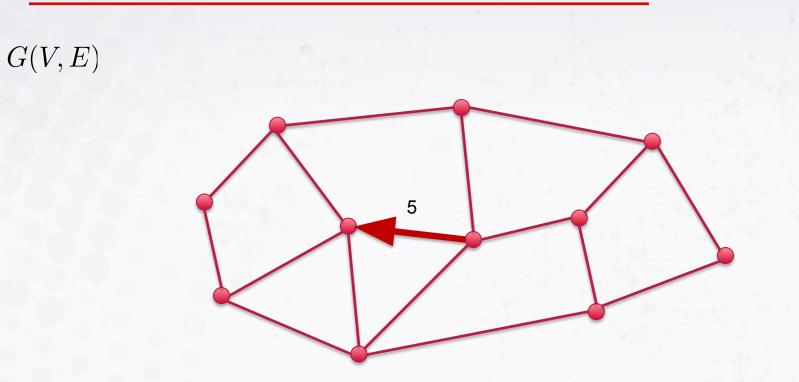
Introducing queues





Network represented as a directed, bi-directional graph G(V, E)



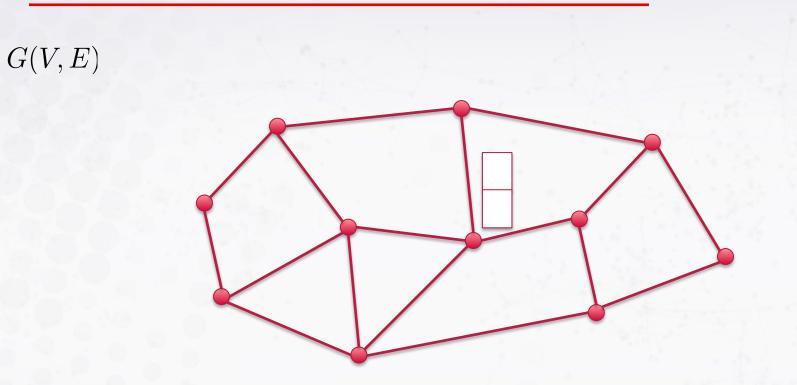


Each edge $(u, v) \in E$ has a cost/weight w_{uv}

[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$



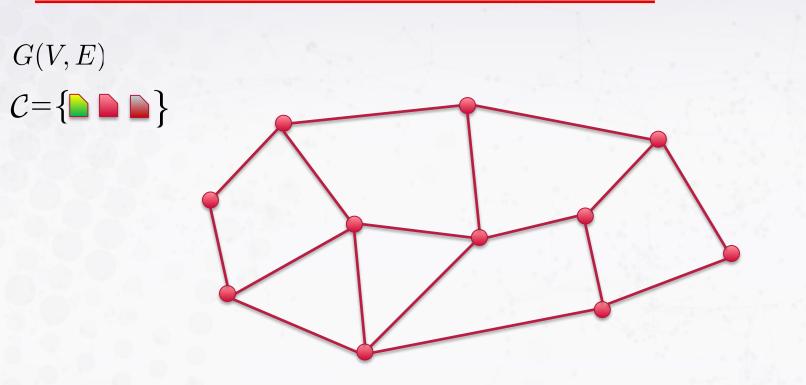


[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$

Node $v \in V$ has a cache with capacity $c_v \in \mathbb{N}$



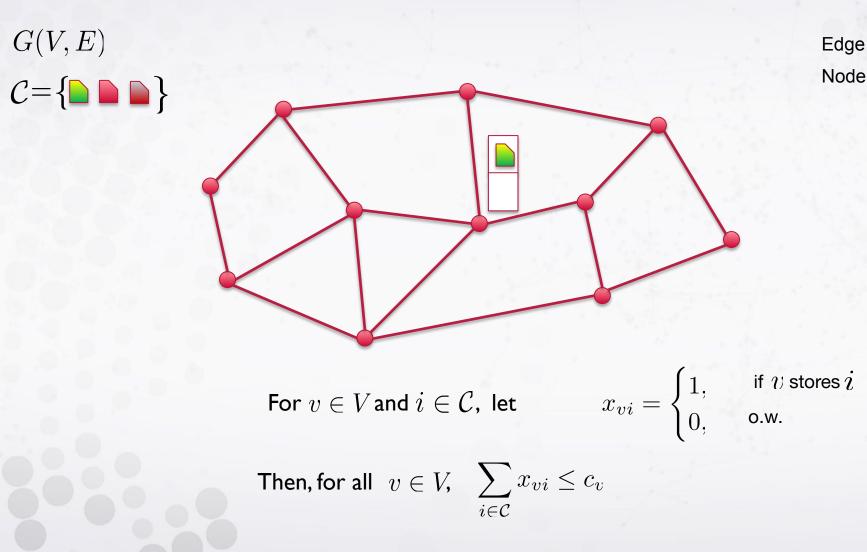


[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$

Items stored and requested form the **item catalog** $\, \mathcal{C} \,$



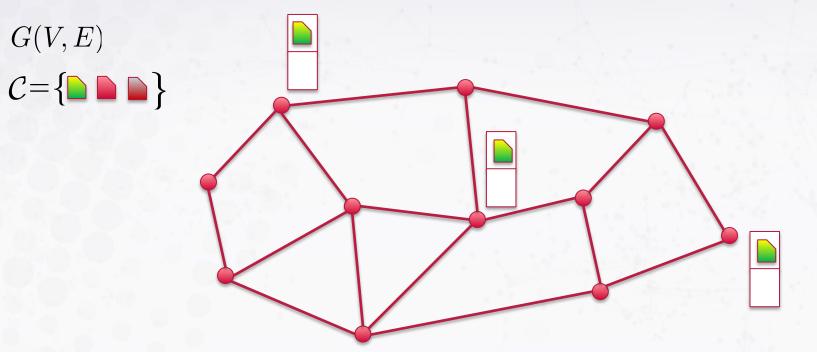


[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$



Model: Designated/Permanent Servers



[I. and Yeh, SIGMETRICS 2016/ToN 2018]

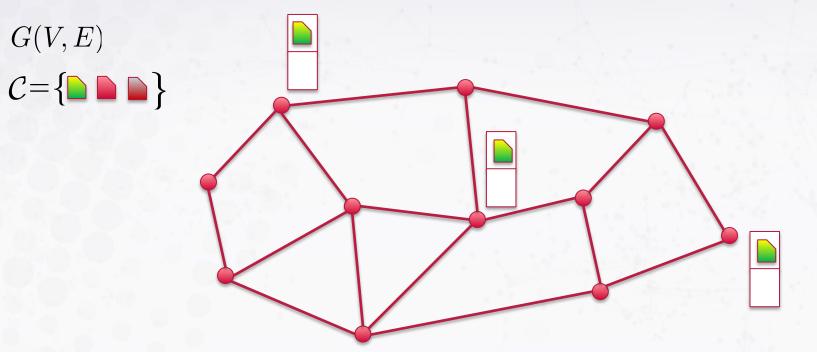
Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$

For each and $i \in C$, there exists a set of nodes $S_i \subset V$ (the **designated servers** of *i*) that **permanently store** *i*.

I.e., if $v \in S_i$ then $x_{vi} = 1$



Model: Designated/Permanent Servers



[I. and Yeh, SIGMETRICS 2016/ToN 2018]

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Model: Demand



Requests are always satisfied!

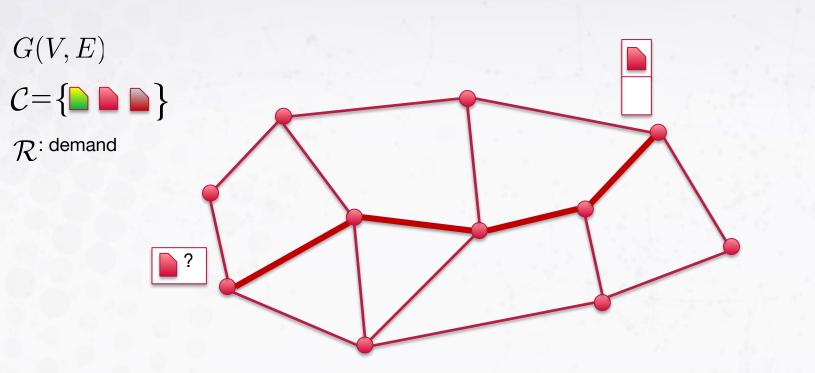
A **request** is a pair (i, p) such that:

 \Box *i* is an item in C

 $\square p = \{p_1, \ldots, p_K\}$ is a simple path in G such that $p_K \in S_i$.



Model: Demand



[I. and Yeh, SIGMETRICS 2016/ToN 2018]

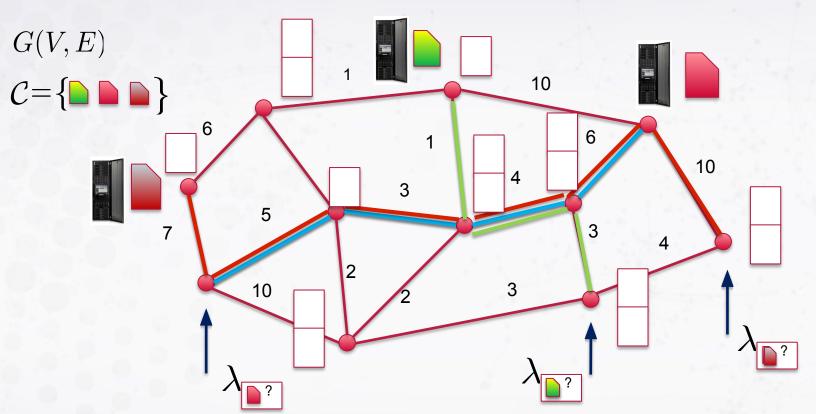
Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$ Request rates: $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$

Demand \mathcal{R} : set of all requests (i, p)

Request arrival process is Poisson with rate $\lambda_{(i,p)}$



Model: Goal

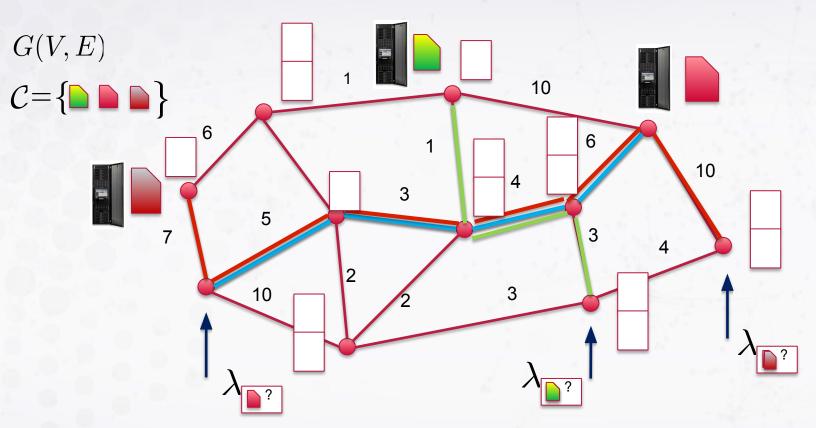


Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$ Request rates: $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$

Design content allocation so that expected transfer costs are minimized.



Model: Goal



Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$ Request rates: $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$

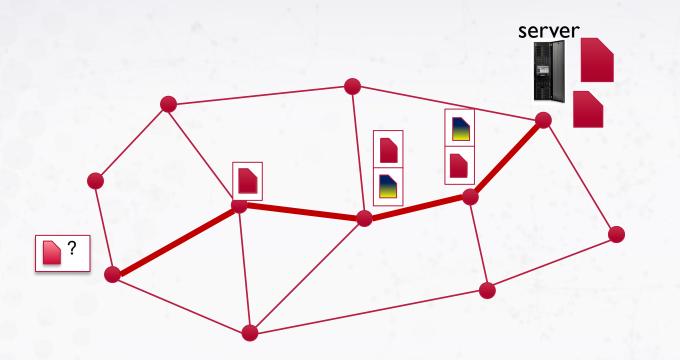
Challenge: Caching algorithm should be

- **adaptive**, and
- distributed.



A Simple Algorithm: Path Replication + LRU

[Cohen and Shenker 2002] [Jacobson et al. 2009]



- ✔ Distributed
- ✔ Adaptive
- ✓ Extremely Popular

- Cache item on every node in the reverse path
 Evict using a simple policy, e.g., LRU, LFU, FIFO etc.
- □ Many variants: Move-Copy-Down (MCD), Leave-Copy-Down (LCD)...



A Simple Algorithm: Path Replication + LRU

	A			
Google	CDN +	CDN + LRU		
	Q All	🗉 News	🖾 Images	► Vid
	About 329,000 results (0.50 seconds)			

CDN Caching Explained – Stac

https://cupport.stackpath.com > 2600

Cache item on every node in the reverse path
 Evict using a simple policy, e.g., LRU, LFU, FIFO etc.

□ Many variants: Move-Copy-Down (MCD), Leave-Copy-Down (LCD)...

- Distributed
- ✔ Adaptive
- ✓ Extremely Popular



[Cohen and Shenker 2002] [Jacobson et al. 2009] But...

Path Replication + LRU is arbitrarily suboptimal.



Path Replication + LRU is Arbitrarily Suboptimal

Cost when caching

 $0.5 \times 1 + 0.5 \times 2 = 1.5$

Cost of PR+LRU:

 $0.25 \times (M+1) + 0.25 \times 1 + 0.25 \times 2 + 0.25 \times 1 = 0.25M + 1.25$

□ When M is large, PR+LRU is **arbitrarily suboptimal!**

 $\lambda_{
ho:2} = \lambda_{
ho:2} = 0.5$ requests per sec

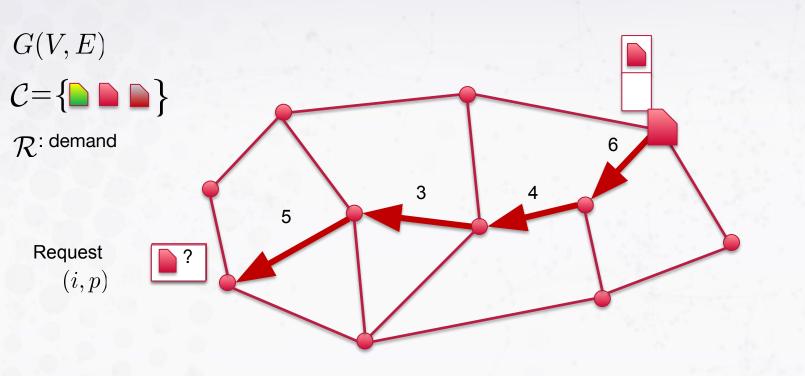
N

True for any strategy (LRU,LFU,FIFO,RR+LCD,MCD) that ignores upstream costs!!



[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Model: Routing Costs & Caching Gain



[I. and Yeh, SIGMETRICS 2016/ToN 2018]

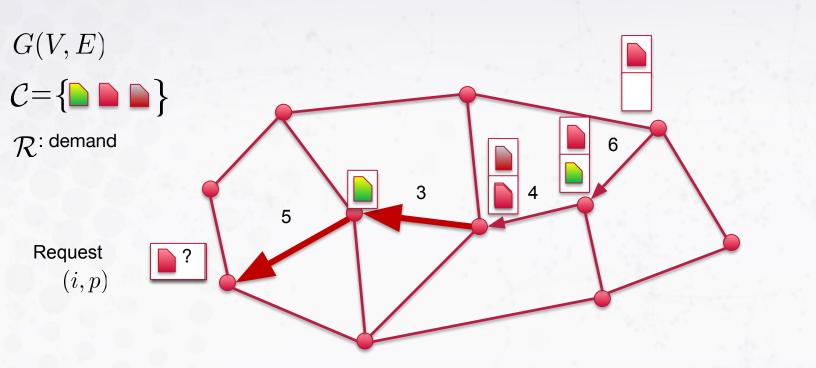
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Worst case routing cost:

18



Model: Routing Costs & Caching Gain



18

8

Worst case routing cost:

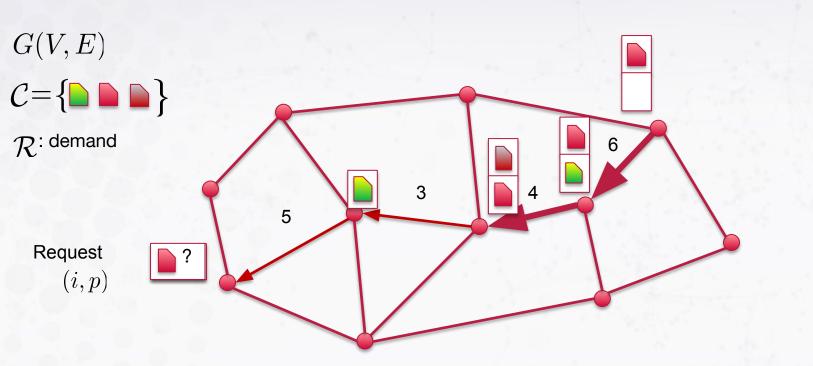
Cost due to intermediate caching:

[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$ Request rates: $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$

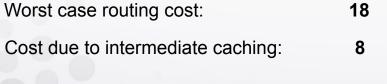
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Model: Routing Costs & Caching Gain



[I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$ Request rates: $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$



Caching Gain:

18-8 = 10



Objective: Maximizing Caching Gain

 $\mathcal{C} = \{ \square \square \square \}$ $\mathcal{R}^{: \text{demand}}$

G(V, E)

 [I. and Yeh, SIGMETRICS 2016/ToN 2018]

Edge costs: $w_{uv}, (u, v) \in E$ Node capacities: $c_v, v \in V$ $\sum_{i \in C} x_{vi} \leq c_v$, for all $v \in V$ Request rates: $\lambda_{(i,p)}, (i,p) \in \mathcal{R}$

Maximize:

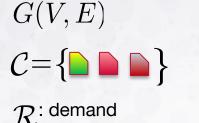
$$MAXCG$$
$$F(X) = \sum_{(i,p)\in\mathcal{R}} \lambda_{(i,p)} \sum_{k=1}^{|p|-1} w_{p_{k+1}p_k} \left(1 - \prod_{k'=1}^k (1 - x_{p_{k'i}}) \right)$$

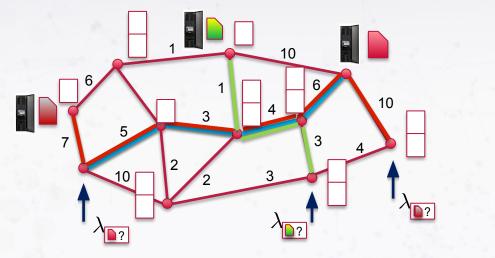
Subject to:

$$\begin{split} \sum_{i \in \mathcal{C}} x_{vi} &= c_v, & \text{for all } v \in V \\ x_{vi} &= 1, & \text{for all } i \in \mathcal{C} \text{ and } v \in S_i \\ x_{vi} &\in \{0, 1\}, & \text{for all } v \in V \text{and } i \in \mathcal{C} \end{split}$$

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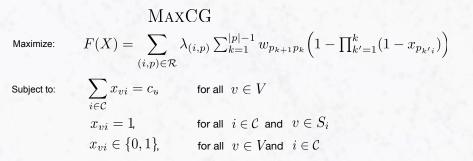
Objective: Maximizing Caching Gain





[I. and Yeh, SIGMETRICS 2016/ToN 2018]

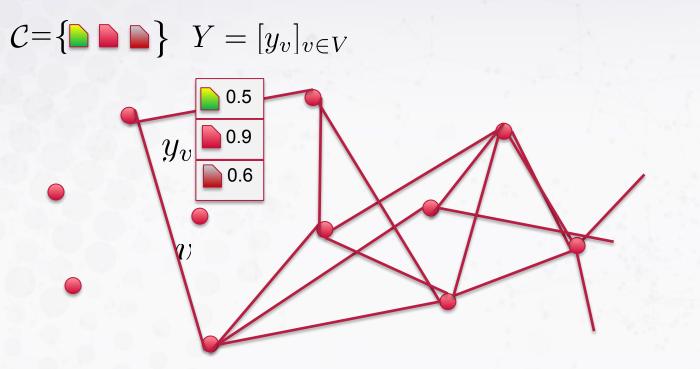
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- □ NP-hard but...
- .. Submodular maximization under matroid constraints
- **1-1/e polytime approximation** algorithm

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Distributed, Adaptive Algorithm

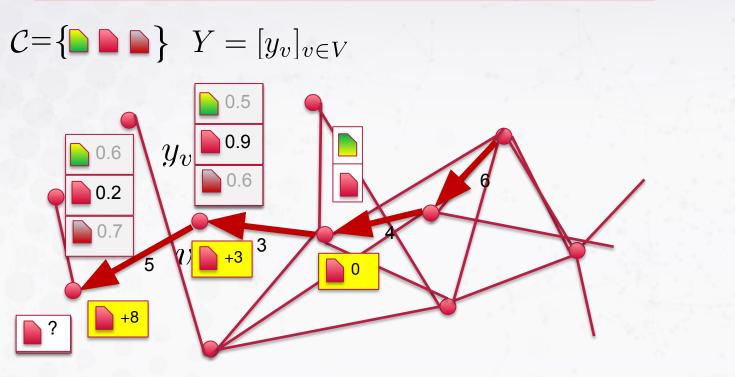


Each cache maintains state
 State = probability of caching item

[I. and Yeh, SIGMETRICS 2016/ToN 2018]



Distributed, Adaptive Algorithm



Theorem: The proposed algorithm leads to an allocation X_k such that

 $\lim_{k \to \infty} \mathbb{E}[F(X_k)] \ge (1 - \frac{1}{e})F(X^*)$

where X^* an optimal solution to the (NP-hard) offline problem.

[I. and Yeh, SIGMETRICS 2016/ToN 2018]

- Each cache maintains state
 State = probability of caching item
- Upon request, control message collects information about upstream costs
 - = gradient of concave relaxation of objective (in expectation)
- During slot of length T, average upstream costs
- At end of slot, adapt state and refresh contents by randomly sampling from distribution/state, independently across nodes.

• **"value"** of item is

 $\lambda_{\mathbb{P}} \times \mathbb{E}[$ upstream cost upon \mathbb{D} miss]



No-Regret Algorithms

- □ Theorem assumes:
- □ Stationary, stochastic request arrivals
- □ Negligible costs for updates



Arbitrary, adversarial request arrivals per time-slot
 Account for update costs

Theorem: A **distributed**, **online** algorithm that attains regret

$$R(T) = (1 - \frac{1}{e}) \sum_{t=1}^{T} F_t(X^*) - \left(\sum_{t=1}^{T} F_t(X_t) - \sum_{t=1}^{T} UC(X_t, X_{t-1})\right) = O(\sqrt{T})$$

optimal offline static policy

caching gain of online policy

penalty for update costs



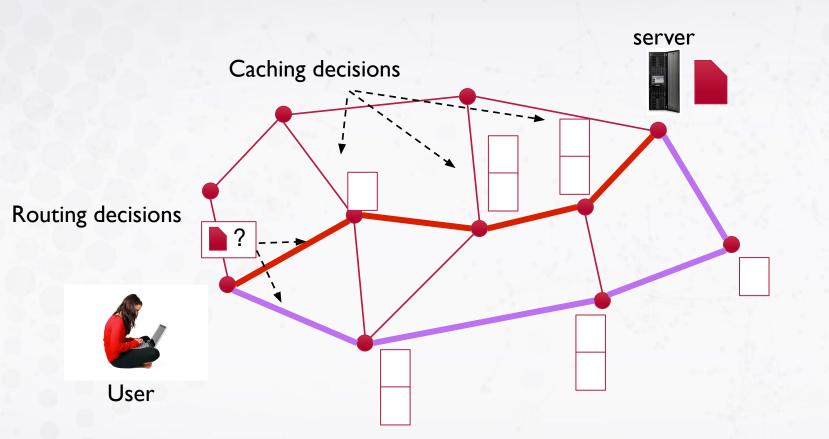
Cache network optimization

□ Jointly optimizing caching and routing

Introducing queues



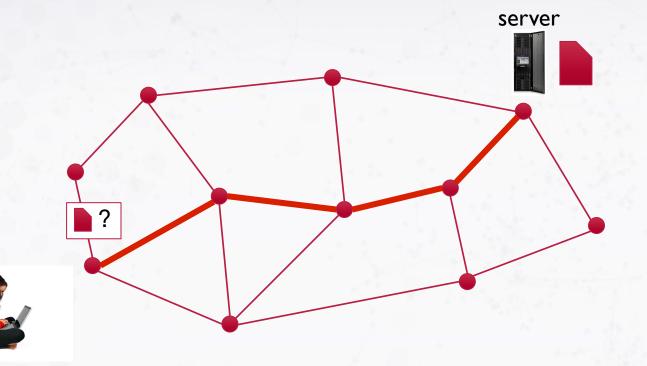
Joint Optimization



Both caching and routing decisions are part of optimization



Is Joint Optimization Really Necessary?



Shortest Weight Path

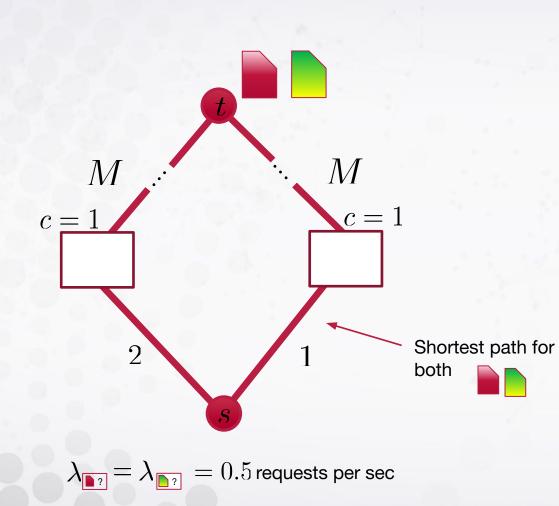
User

Why not just use **shortest weight path** routing towards **nearest designated server**?



Shortest Path Routing is Arbitrarily Suboptimal

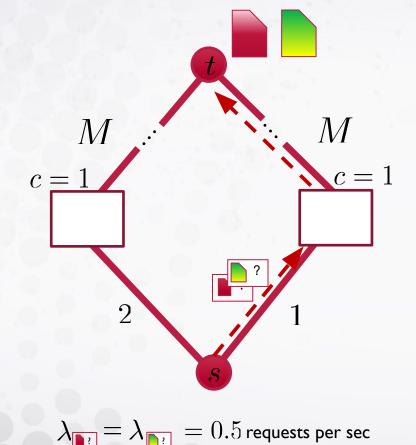
[I. and Yeh, ICN 2017/JSAC 2018]





Shortest Path Routing is Arbitrarily Suboptimal

[I. and Yeh, ICN 2017/JSAC 2018]



 $\lambda_{ ?}$

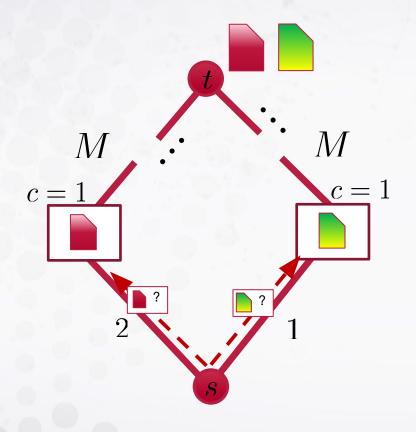
 $-\lambda_{\uparrow}$

Irrespective of caching algorithm used, cost under shortest path routing is $\Theta(M)$



Shortest Path Routing is Arbitrarily Suboptimal

[l. and Yeh, ICN 2017/JSAC 2018]



 $\lambda_{
horevect}=0.5$ requests per sec

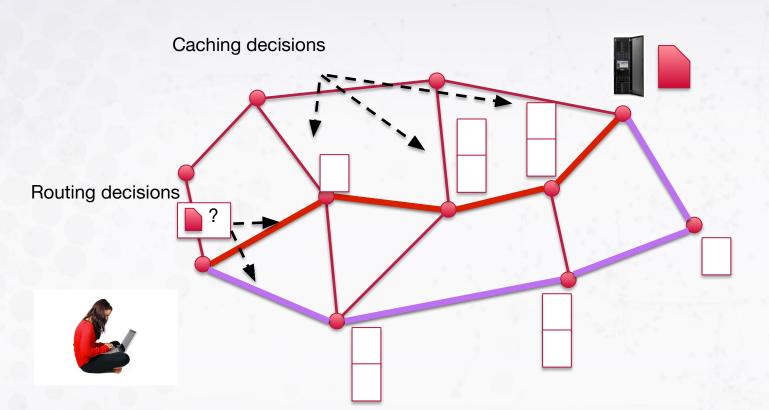
Irrespective of caching algorithm used, cost under shortest path routing is $\Theta(M)$

Cost under "split" routing strategy is O(1).

Shortest path routing to nearest server is arbitrarily suboptimal.



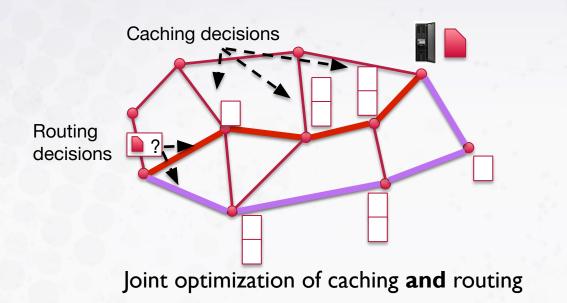
Key Intuition



Increasing **path diversity** creates **more caching opportunities**.



Algorithms with Guarantees



Stochastic requests

[I. and Yeh, ICN 2017/JSAC 2018]

- Distributed, adaptive algorithm within
 I-I/e from the optimal
- Adversarial requests [Li, Si Salem, Neglia, and I., SIGMETRICS 2022]
 - **Distributed**, online algorithm with $O(\sqrt{T})$ regret w.r.t. I-I/e from the optimal offline solution



Experiments

Graph Topologies

Graph	V	E	$ \mathcal{C} $	$ \mathcal{R} $	c_v	$ \mathcal{P}_{(i,s)} $
cycle	30	60	10	100	2	2
grid-2d	100	360	300	1 K	3	30
hypercube	128	896	300	1 K	3	30
expander	100	716	300	1 K	3	30
erdos-renyi	100	1042	300	1 K	3	30
regular	100	300	300	1 K	3	30
watts-strogatz	100	400	300	1 K	3	2
small-world	100	491	300	1 K	3	30
barabasi-albert	100	768	300	1 K	3	30
geant	22	66	10	100	2	10
abilene	9	26	10	90	2	10
dtelekom	68	546	300	1 K	3	30

Routing Algorithms

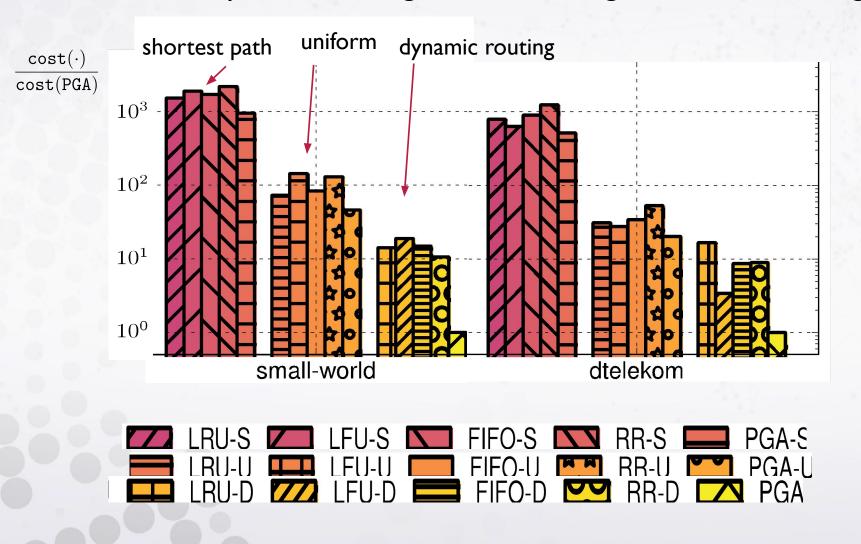
- □ Shortest Path Routing
- Uniform
- Dynamic routing: PGA on L for routes alone

Caching Algorithms

LRU
LFU
FIFO
RR
PGA on L

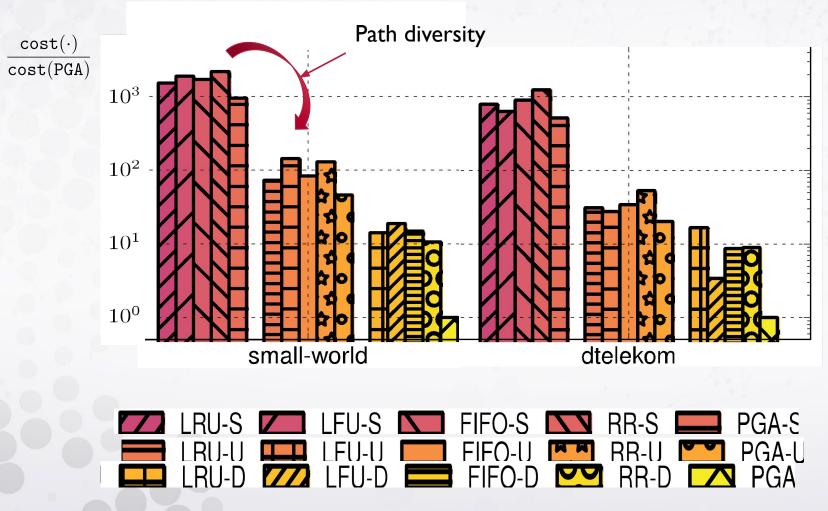


Ratio of expected routing cost to routing cost under our algorithm



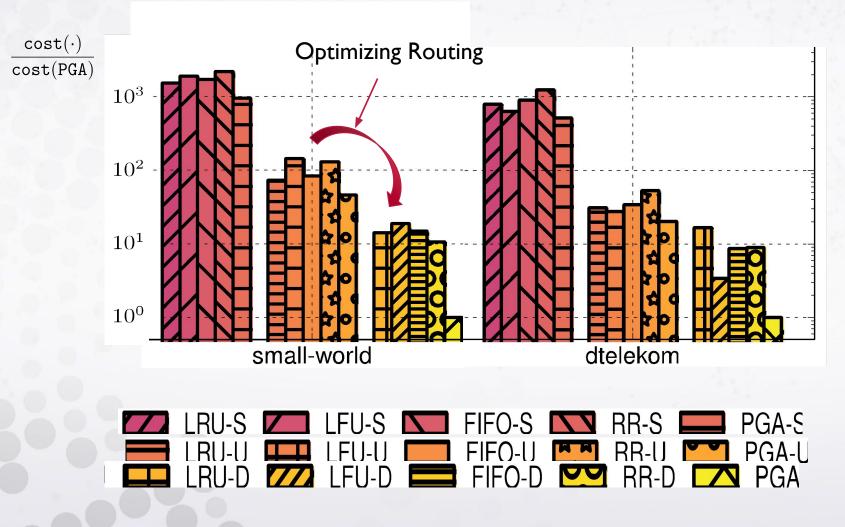


Ratio of expected routing cost to routing cost under our algorithm

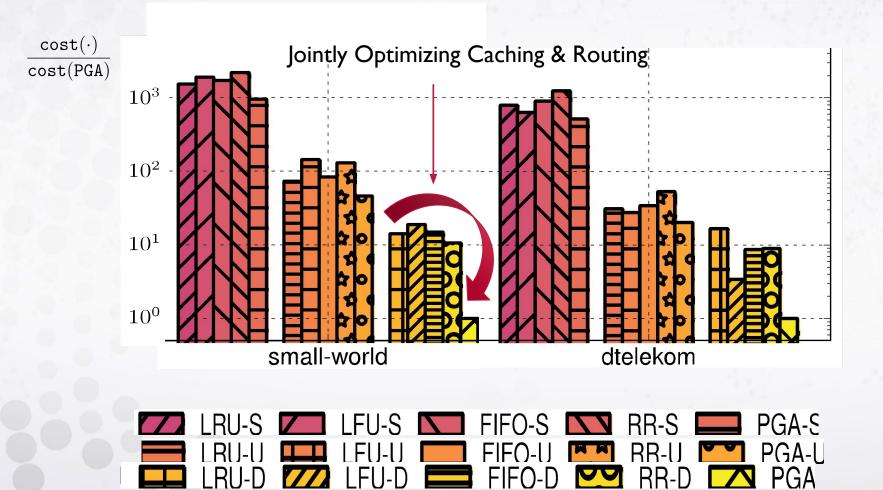




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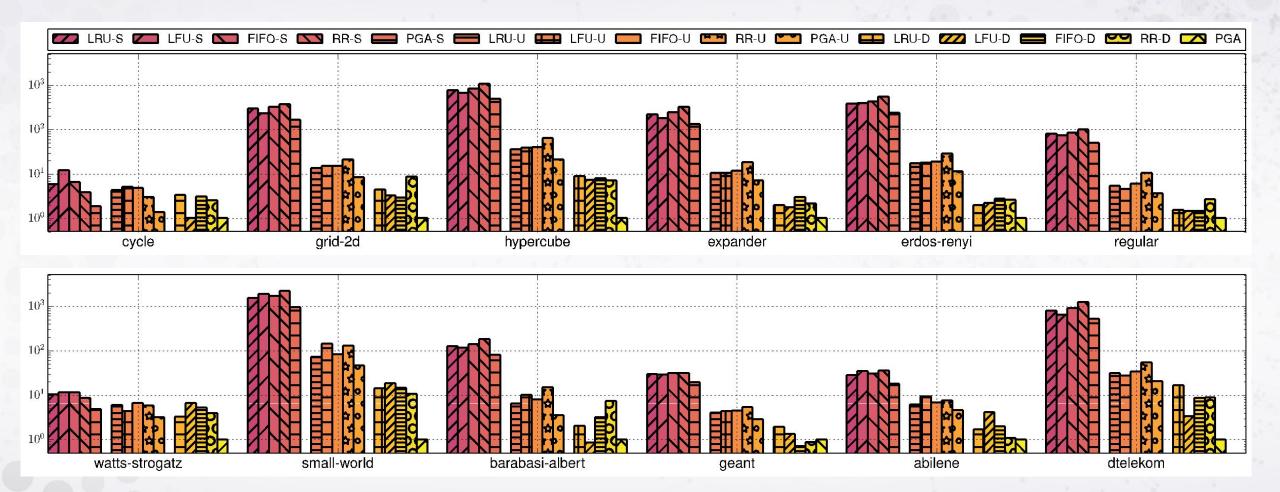








[I. and Yeh, ICN 2017/JSAC 2018]





Overview

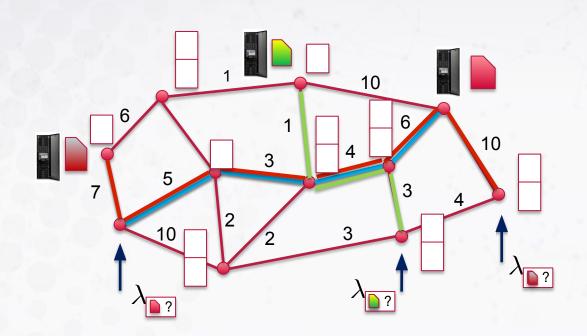
Cache network optimization

Jointly optimizing caching and routing

Introducing queues

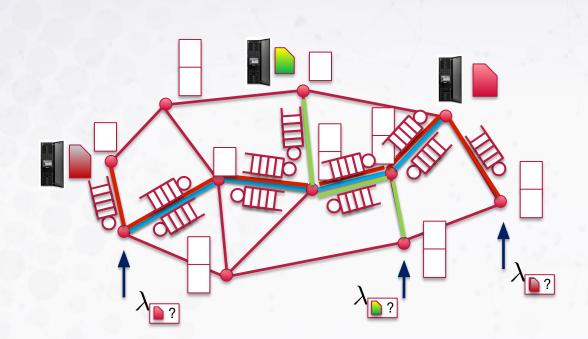


Introducing Queues





Introducing Queues



- Downward edges are associated with M/M/1 queues
- Determine cache contents so that steady state queuing costs are minimized
- \Box Size of queue at edge $e: n_e \in \mathbb{N}$
- \Box Cost: $c_e(n_e)$ where $c_e: \mathbb{R}_+ \to \mathbb{R}_+$ is non-decreasing
 - □ Queue size, its moments, queuing delay, occupancy probability...
- □ Aggregate expected cost:

$$C(\mathbf{x}, \boldsymbol{\lambda}) \equiv \sum_{e \in E} \mathbb{E}_{\mathbf{x}, \boldsymbol{\lambda}}[c(n_e)]$$

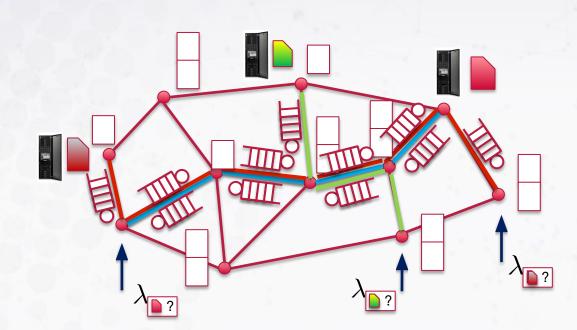
$$F(\mathbf{x}, \boldsymbol{\lambda}) = C(\mathbf{x}_0, \boldsymbol{\lambda}) - C(\mathbf{x}, \boldsymbol{\lambda})$$

caching allocation under which system is stable

Theorem: Maximizing caching gain is a submodular maximization problem subject to matroid constraints.



Stability



Caching gain:

$$F(\mathbf{x}, \boldsymbol{\lambda}) = C(\mathbf{x}_0, \boldsymbol{\lambda}) - C(\mathbf{x}, \boldsymbol{\lambda})$$

caching allocation under which system is stable

□ How does one find this?

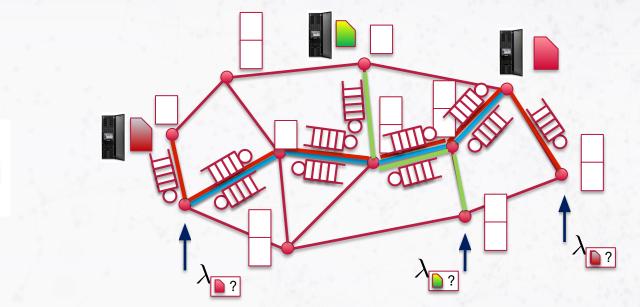
- Optimize caching strategy (\mathbf{x}) and jointly do admission control (λ) [Kamran, Moharrer, I., and Yeh, INFOCOM 2021] subject to stability constraints.
 - Much weaker optimality guarantees.



A More Elegant Solution: Counting Queues

[Li and I., INFOCOM 2020/ToN 2021]







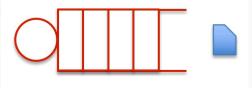
M/M/1 queue



A More Elegant Solution: Counting Queues

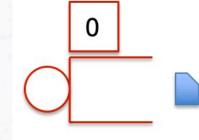
[Li and I., INFOCOM 2020/ToN 2021]

Identical responses merge when collocated



M/M/1 queue

 $\mathcal{C} = \{ \square \square \square \}$

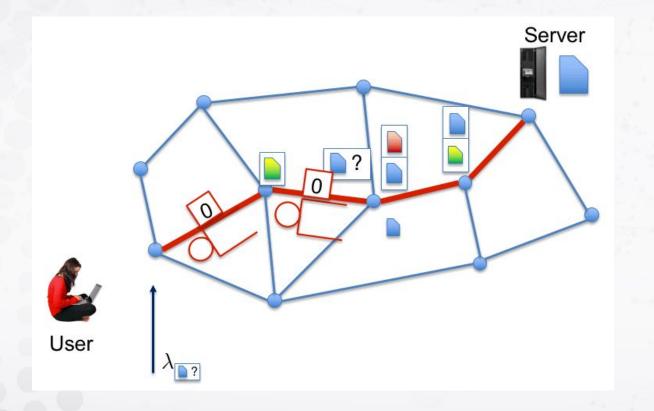


M/M/1c queue



A More Elegant Solution: Counting Queues

[Li and I., INFOCOM 2020/ToN 2021]



- □ Network with **counting queues**
- Not reversible, steady-state queue distribution has no closed form
- □ Well-approximated by M/M/∞ queues
- Theorem: Under this approximation, there exists an algorithm jointly optimizing of caching and service rate allocations within 1-1/e of the optimal.



No-regret algorithms

Merging requests/queries, not responses

Joint optimization tasks

- Caching
- Routing
- Service assignment
- Admission control

Departure from submodularityDistributed algorithms





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Adaptive Caching Networks with Optimality Guarantees S. Ioannidis and E.Yeh, SIGMETRICS 2016/ToN 2018.

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Online Caching Networks with Adversarial Guarantees Y. Li, T. Si Salem, G. Neglia, and S. Ioannidis, SIGMETRICS/PERFORMANCE 2022.



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Thank You!